Tutorial 0

TA information: Haiyu CHEN. hychen@math.cuhk.edu.hk.

The following answer is author's opinion on subjects and could be bias.

1. Organization of tutorial.

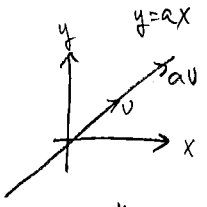
- Hightlight important things in lecture
- work out examples or sample problems
- answer questions.
- ANYTHING YOU PROPOSE!

Do not hasitate to let me know your advice! I will change the style to suit your need!

2. What is linear algebra?

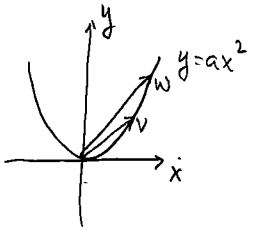
Algebra: $+, -, \times, \div$, other operations

linear: line



1' $v \in V$, $av \in V$ · scaling.

2' $v, w \in V$, $v+w \in V$ addition



$v+w \notin \cup$

$av \notin \cup$

not linear.

This is studied in algebraic geometry.

3. MATH1030 V.S. MATH2040?

1030 concrete vectors in $\mathbb{R}^n$, $\mathbb{C}^n$, matrices.

2040
- abstraction of 1030 so that it has wider applications!
- A vector is anything that satisfy properties 1', 2' above.
- so we don't need to limit ourselves to $\mathbb{R}^n$, $\mathbb{C}^n$, $M_n(\mathbb{F})$.
- We can do linear algebra as long as we have a field $\mathbb{F}$ and things that behaves like a vector.

4. What does the subject linear algebra consist of (for a first course)?

Linear algebra

- ① Vector spaces V.
  - Classification of vector spaces? — dimension — basis
    - span
    - Linear independence
  - ↑ What are the objects of a given type, up to some equivalence?
- ② Maps between them $V \to V$.
  - Classification of linear transformations?
    - invertibility
    - Jordan normal form, Jordan decomposition.
    - primary decomposition
      - eigenvalue
      - eigen space, Cayley-Hamilton
      - (eigenvalue, eigen space) > diagonalizability.
    - matrix representation — 1130.
  - kernel, range, rank-nullity thm.
- ③ inner product spaces
  - notion of angle, length?
    - orthogonality
    - Gram-Schmidt process
- ④ interactions between ② and ③.
  - adjoint operators ——— matrix transpose
  - normal, self adjoint, unitrary operators — symmetry, unitary matrices.
  - Spectrum theorem: diagonalization of above linear operators.
- ⑤* Bilinear forms, quadratic forms and multilinear algebra.
- Applications to later courses:
  - Topology of manifold
  - Functional analysis
  - Abstract algebra
  - Modules and representation theory.
  - applied math, physics, computer science etc.

The dictionary

These connections are rough connections, or generalizations.

2040	1030
vector space	vectors
linear combination	column space
dimension	rank
standard basis	reduced echelon forms
linear maps	matrices
kernel	null space
range	column space
composition	matrix multiplication.
invertible maps	invertible matrices
change of basis	similar matrices.
orthonormal basis	orthonormal matrices
eigenspace	eigenvector, eigenspace.
norm, inner product	length, angle.
invariant space	block matrices
adjoint operator	matrix transpose
self-adjoint operator	symmetric matrices. Hermitian matrices
spectral decomposition	congruent matrices.